OCR
RECOGNISING ACHIEVEMENT

GCE

Advanced GCE

Unit **4735:** Probability and Statistics 4

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
AEF	Any equivalent form

Subject-specific Marking Instructions for GCE Mathematics (OCR) Statistics strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not **or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awar

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark s designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full m must not be judged on the answer alone, and answers that are given in the question, especially, must be validly steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* inco Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the m award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if severa candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially under marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually suffici candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more workin ana establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this w case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is use that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no m sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, t are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previous results. Otherwise A and B marks are given for correct work only — differences in notation are of course permit (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded f intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, e acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leade

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A mar 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candida question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicat

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decim often be appropriate. But even this does not always apply – quotations of the standard critical points for significa as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to whi wer is example, if 3 significant figures are expected (either because of an explicit instruction or because th l co problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; bu are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas expected) should not be penalised. However, answers which are *grossly* over- or under-specified should norm loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.1 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to a given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greate accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should b the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, th should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what a the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/o the question being considerably changed, it is likely that all the marks for that question, or section of the questio However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases a below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread o (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.67 slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and shou if eve
2. Commonly in sections of questions where there is a numerical answer either at the end of the se be
commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be
[correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this v
must be withheld even if all method marks have been earned. It will also often be the case that such a mar
even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow
candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking sc
the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fres
not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expre
misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but
they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also a
misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detaile
should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	G
1	(i)		$F = 1$, $S = 1$ requires HTH or THT Probability = $^1/_8 + {}^1/_8 = {}^1/_4$ AG	M1 A1 **[2]**	Clear method – not just multiplica SC $\frac{2}{8} = \frac{1}{4}$ ONLY seen B1. NOT $\frac{1}{2} \times \frac{1}{2}$
1	(ii)		Marginals : 0 1 2 p(S) ¼ ½ ¼ p(F) ¼ ½ ¼ E(S) = 1×½ + 2×¼ = 1 = E(F) E(SF) = $\frac{1}{4} + \frac{2}{8} + \frac{2}{8} + \frac{4}{8} (= 1\frac{1}{4})$ Cov(S,F) = E(SF) – E(S)E(F) = ¼	M1 A1 B1 M1* *M1 A1 **[6]**	Correct method, can be implied by (i) Both correct. Can be implied by e.g. E(S) = or symmetry.
2			H_0: $m_{II–I} = 0$, H_1: $m_{II–I} < 0$ II–I: 1, –14, –9, 2, –7, –8, 3, –4, –10, 6, 5, 13 Rank: 1, –12, –9, 2, –7, –8, 3, –4, –10, 6, 5, 11 $P = 1 + 2 + 3 + 6 + 5 = 17$ $Q = 61$ so $T = 17$ 5% CR: $T \leq 17$ T is inside CR so reject H_0 There is sufficient evidence at the 5% SL that drug II is associated with fewer sneezes	B1 M1 A1 M1A1 M1 A1 **[7]**	Allow $m_1 > m_2$, $m_d > 0$, etc. or in words, bu of parameters or population ft TS & CV ft TS only. Contextualised, not over-asserti

Question			Answer	Marks	
3	(i)		$M(t) = \int_0^\infty \frac{1}{4}xe^{-\frac{1}{2}x(1-2t)}dx$ oe	M1*	From $E(e^{xt})$. Need single expone
			$= \left[\frac{-xe^{-\frac{1}{2}x(1-2t)}}{2(1-2t)}\right]_0^\infty + \int_0^\infty \frac{e^{-\frac{1}{2}x(1-2t)}}{2(1-2t)}dx$	*M1A1	Integration by parts
			$= \left[\frac{-e^{-\frac{1}{2}x(1-2t)}}{(1-2t)^2}\right]_0^\infty$	A1	$= \left[\frac{-e^{-\frac{1}{2}x(1-2t)}}{4(t-\frac{1}{2})^2}\right]_0^\infty$. Allow without limits.
			$= AG \quad (1-2t)^{-2}$	A1	With evidence, cwo
			Requires $1 - 2t > 0$ for correct limits	B1	Or for convergence of the integral
				[6]	
3	(ii)		$M'(t) = 4(1-2t)^{-3}$ $E(X) = 4$ cwo	B1	or from $1 + 4t$
			$M''(t) = 24(1-2t)^{-4}$ $E(X^2) = 24$ cwo	B1	$+ 12t^2$
			Var = 24 – 16 = 8	B1FT	provided Var > 0.
				[3]	
4	(i)		Distribution of heights may not be normal/is unknown	B1	Allow "No assumption required", but nothi
				[1]	Not "groups independent" unless somethin
4	(ii)		H_0: $m_A = m_B$, H_1: $m_A \neq m_B$	B1	Medians. Allow words in context. Not μ ur
			Ranks:		
			A: 4, 8, 10, 11, 14, 15, 16, 18, 20, 21, 22	B1	
			B: 1, 2, 3, 5, 6, 7, 9, 12, 13, 17, 19		
			$m = n = 11$, $R_m = 159$ or 94	B1	
			Use normal approximation with mean 126.5 [= 253/2]	M1	allow $\frac{1}{2} \times 11 \times (11 + 11 + 1)$
			Variance 231.92 [= 2783/12]	B1	allow $\frac{1}{12} \times 11 \times 11 \times (11 + 11 + 1)$
			(α) $P(\leq 94) = \Phi((94.5 - 126.5)/\sqrt{(231.92)})$	M1	Standardising. Allow no/incorrect cc.
			or $P(\geq 159) = 0.0178$	A1	Value
			< 0.025 and reject H_0	M1	ft TS
			(β) $z = (94.5 - 126.5)/\sqrt{(231.92)} = -2.101$	M1A1	Standardising ; value
			< -1.96 so reject H_0	M1	ft TS
			There is evidence that salinity affects growth	A1	Or equivalent in context. ft TS.
				[9]	

Question			Answer	Marks	
5	(i)		$G(t) = E(t^X) = {}^1/_{16}(1 + 4t + 6t^2 + 4t^3 + t^4)$ $= {}^1/_{16}(1 + t)^4$	M1A1 A1 **[3]**	Correct form ; correct coefficie allow $(\frac{1}{2} + \frac{1}{2}t)^4$.
5	(ii)		$G'(t) = {}^1/_4(1 + t)^3$ $E(U) = G'(1)$ **= 2** $G''(t) = {}^3/_4(1 + t)^2$ $Var(U) = G''(1) + G'(1) - (G'(1))^2$ $= 3 + 2 - 4 =$ **1**	M1 A1 M1 A1 **[4]**	or expanded form. No marks from part (i Finding G'' and formula correct
5	(iii)		B(4, ½)	B1 B1 **[2]**	Binomial Parameters
5	(iv)		$Y = 0 \quad 1 \quad 4 \quad 9 \quad 16$ $G_Y(t) = {}^1/_{16} + {}^1/_4 t + {}^3/_8 t^4 + {}^1/_4 t^9 + {}^1/_{16} t^{16}$	B1 B1 **[2]**	Values of Y
5	(v)		No, U and Y are not independent	B1 **[1]**	
6	(i)		$E(T_1) = 2E(X) + 3E(Y)$ $= 8\mu$ Unbiased estimate $= (X_1 + X_2 + Y_1 + Y_2 + Y_3)/8$	M1 A1 A1 **[3]**	 NOT $\dfrac{2x + 3y}{8}$
6	(ii)		$E(T_2) = 2\mu + 6c\mu = \mu$ $\Rightarrow c = -{}^1/_6$	M1 A1 **[2]**	Setting up an equation

Question			Answer	Marks	
6	(iii)		$\text{Var}(T_1) = (2\sigma^2+9\sigma^2)/64$ $= {}^{11}/_{64}\sigma^2$ $\text{Var}(T_2) = \sigma^2 + \sigma^2 + {}^{1}/_{36}(3\sigma^2 + 3\sigma^2 + 3\sigma^2) = {}^{9}/_{4}\sigma^2$ T_1 has the smaller variance so is more efficient	M1 A1 A1 A1ft **[4]**	Using var of sum = sum of var
6	(iv)		$E(T_3) = a(2\sigma^2 + 2\mu^2) + b(9\sigma^2 + 12\mu^2) = \sigma^2$ Coefficient of $\mu^2 = 0$ gives $2a + 12b = 0$ Coefficient of $\sigma^2 = 1$ gives $2a + 9b = 1$ Solve to give $a = 2$ and $b = -{}^{1}/_{3}$	M1A1 B1 A1 **[4]**	$(\text{Var}(X) =) E(X^2) - [E(X)]^2$ seen or implied either equation.
7	(i)		$P(A) = P(K)\times 1 + P(K')\times {}^{1}/_{n}$ $= p + (1-p)/n$ $= \dfrac{q+np}{n}$ AG	M1 A1 B1 **[3]**	 allow $p + \dfrac{q}{n}$
7	(ii)		$P(K \cap A) = p$ $P(K\|A) = \dfrac{p}{\frac{q+np}{n}}$ $= \dfrac{np}{q+np}$	B1 M1 A1 **[3]**	 AEF
7	(iii)	(a)	If X answers are correct $100 - X$ are incorrect so score $= 2X - 100 = 40$ giving $X = 70$	B1 **[1]**	70 seen

Question			Answer	Marks	
7	(iii)	(b)	P(A) = 5/8		
			(α) E(X) = $100 \times {}^{5}/_{8} = 62.5$	B1	
			Var(X) = $s^2 = 100 \times {}^{5}/_{8} \times {}^{3}/_{8}$ (= 23.4375) $(=\frac{375}{16})$	M1A1	Allow M1 from wrong p
			P($X \geq 70$) = 1 – Φ[(69.5 – 62.5)/s]	M1A1	Normal approximation. Allow M1 from 4
			= 0.0741	A1	Standardise M1 only if no or wrong cc, A1
			(β) E($2X$ – 100) = 25	B1	
			Var($2X$ – 100) = 93.75	M1A1	
			P($2X$ –100 ≥ 40) = 1 – Φ[(39 – 25)/√(93.75)]	M1A1	Standardise, M1 only for no or wrong cc, A
			= 0.0741	B1	
			(γ) Score per question = S		
			E(S) = $1 \times {}^{5}/_{8} - 1 \times {}^{3}/_{8} = ¼$	B1	
			Var(S) = $1^2 \times {}^{5}/_{8} + 1^2 \times {}^{3}/_{8} - ({}^{1}/_{4})^2$	M1A1	
			Total, $T \sim$ N(25, 93.75)		
			P($T \geq 40$) = 1 – Φ[39 – 25)/√(93.75)]	M1A1	As for β
			= 0.0741	B1	
				[6]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553